Bremsstrahlung in the gravitational field of a global monopole

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We investigate the radiation emitted by a uniformly moving charged scalar particle in the space-time of a point-like global monopole. We calculate the total energy radiated by the particle and the corresponding spectrum, for small solid angle deficit. We show that the radiated energy is proportional to the cube of the velocity of the particle and to the cube of the Lorenz factor, in the non-relativistic and ultra-relativistic cases, respectively.

I. INTRODUCTION

Topological defects may arise in gauge models with spontaneously symmetry breaking. They can be of various types as monopoles, domain walls, strings and their hybrids [1,2]. They are cosmological objects which could be formed during phase transitions in the early universe. They have attracted much attention because of their peculiar properties, space-time structure and possible astrophysical implications. Their nature depends on the topology of the vacuum manifold of the field theory under consideration.

Among the topological defects mentioned previously, in this paper we will focus our attention on global monopoles. The simplest model that gives rise to a global monopole is described by a system composed of a triplet of isoscalar fields whose original global *O(3)* gauge symmetry was spontaneously broken down to *U(1)*.

The gravitational field of global monopoles may lead to the clustering in matter and they can induce anisotropies in the cosmic microwave background radiation. They do not represent any problem for cosmology and for this reason we may proceed in studying their physical consequences. On the other hand, in order to have an agreement with observations, the density of monopoles has to be very low. The first estimation of the density of monopoles was made by Hiscock in Ref. [3] who show that the upper bound on the number density of the global monopoles is at most of one monopole in the local group of galaxies. This estimation was made using the fact that global monopoles produce enormous tidal acceleration which may be important from the cosmological point of view. The subsequent numerical simulations made by Bennet and Rhie show that the upper bound on the density is smaller than that given by Hiscock by many orders [4]. In fact, one has a scaling solution with a few global monopoles per horizon volume. This result was recently recovered by Yamaguchi in Ref. [5].

Recent observations on the cosmic microwave background anisotropy in BOOMERANG [6] and MAXIMA [7] experiments present two peaks in anisotropy, the first is at $l \sim 200$ and the second one at $l \sim 550$ which are, at the first sight, not consistent with the locations and width of peaks expected from topological defects. But as it was noted in Ref. [8] this statement is partly misleading. The amplitude of the Döppler peaks may be good explained in a hybrid model, namely, a model which combines inflation with topological defects. At this moment there is no contradiction with observation data and there is no observation data which permits rule out definitively the possibility of existence of topological defects. Therefore, it is still important from the cosmological as well as from the astrophysical point of view to investigate different kind of effects produced in topological backgrounds.

In the framework of Quantum Electrodynamics, the bremsstrahlung process corresponds to the emission of radiation by a charged particle when it changes its momentum in collision with obstacles such as other particles or when it is accelerated due to the presence of electromagnetic fields. Therefore, in flat space-time particles moving freely do not radiate. On the other hand, in curved space-time the situation is quite different, and in this case, a charged particle moving on geodesic does radiate. This corresponds to the bremsstrahlung process produced by gravitational fields

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and this may arises due to the curvature, topology or due to the combined effects of the geometric and topological features of the space-time.

Alongside with the above radiation emitted by freely moving particles there is another process leading to radiation. It is well known that a charged particle placed in a curved space-time, even at the rest, experiences a self-force due to the geometrical and topological features of the space-time. In particular, for a conical space-time, it is entirely due to the non-local structure of the gravitational field [9]. In the space-time of a point-like global monopole this self-force has already been calculated [10] and it is due to the combined effects of the geometry and topology of this space-time. In this case, the self-potential has the Coulomb structure and therefore the particle is repelled by the monopole. Due to this acceleration the particle emits radiation in a standard way.

In this paper we consider the problem concerning the emission of radiation by a freely moving particle, caused by the combined effects of the geometrical and topological features of the space-time generated by a point-like global monopole. A similar problem has already been considered in Refs. [11–13] in the context of infinitely thin cosmic strings. It was shown that even freely moving particles in this space-time emit radiation. The origin of this radiation is associated with the conical structure of the cosmic string space-time which produces an effect which is proportional to the angle deficit. It is worth to mention that this process is forbidden in the empty Minkowski space-time due to the energy conservation law.

To begin with, let us first introduce the solution corresponding to a global monopole considered by Barriola and Vilenkin [14]. They considered the simplest model that give rise to a global monopole which is described by the Lagrangian

$$L = \frac{1}{2}(\partial_\mu \phi^a)(\partial^\mu \phi^a) - \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2, \tag{1}$$

where ϕ^a is a triplet of self-coupling scalar fields and η is the symmetry-breaking scale.

Combining this matter field with the Einstein equations and considering the general form of the metric with spherical symmetry

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2}$$

the gravitational field is solved and gives the following result

$$B = A^{-1} = 1 - 8\pi\eta^2 - 2\frac{M}{r}, \tag{3}$$

where $M \sim M_{core}$. It is worth noticing that far away from the global monopole core the main effects are produced by the solid angle deficit and thus we can neglect the monopole's mass. Therefore, we obtain the metric of a point-like global monopole which can be written as

$$ds^2 = \alpha^2 dt^2 - \alpha^{-2}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{4}$$

where the parameter α is connected with the energy scale of symmetry breaking η and is given by the relation $\alpha^2 = 1 - 8\pi\eta^2$. For typical grand unified theory the parameter η is of order $10^{16} GeV$ and thus $1 - \alpha^2 = 8\pi\eta^2 \sim 10^{-5}$. The space-time (4) is the solution of Einstein equations with diagonal energy momentum tensor with components $T^\mu_\nu = diag(2,2,1,1)(\alpha^2 - 1)/r^2$.

Rescaling the time and radial coordinates by relations $t \to t/\alpha^2$ and $r \to r\alpha^2$ we obtain the following form for the line element

$$ds^2 = dt^2 - dr^2 - \alpha^2 r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{5}$$

which will be used in what follows. Here $r \in [0,\infty],\ \theta \in [0,\pi],\ \varphi \in [0,2\pi]$.

This metric corresponds to a space-time with a deficit solid angle $\Delta = 32\pi^2 G\eta^2$; test particles are deflected (topological scattering) by an angle $\pi\frac{\Delta}{2}$ irrespective to their velocity and impact parameter. In spite of having constant coefficients g_{00} and g_{rr}, this metric represents a curved space-time whose curvature vanishes in the case $\alpha = 1$ (flat space-time). For $\theta = \frac{\pi}{2}$, the metric (5) is exactly the same as that of a gauge cosmic string, in which case the azimuthal angle φ has a deficit $\Delta = 2\pi(1-\alpha)$.

Therefore, the gravitational field of a global monopole exhibits some interesting properties, particularly those concerning the appearance of nontrivial space-time topologies.

This paper is organized as follows. In Sec. II we find an expression for the spectrum and total energy and analyze these results in two limits: non-relativistic and ultra-relativistic cases. In Sec III we end up comparing the energy radiated by a freely moving particle and by an accelerated one due to the self-force and presenting some conclusions.

Throughout this paper we use units $c = G = 1$.

II. THE ENERGY AND SPECTRUM OF RADIATION.

Now, let us consider a scalar particle with scalar charge q living in this space-time. The scalar and minimal coupling field corresponding to this particle obeys the Klein - Gordon equation

$$\Box\Phi(x) = -4\pi j(x), \tag{6}$$

with a scalar current

$$j(x) = q\int \delta^4(x - x(\tau))\frac{d\tau}{\sqrt{-g}} = \frac{q}{u^0}\frac{\delta(r - r(t))\delta(\varphi - \varphi(t))\delta(\theta - \theta(t))}{\alpha^2 r^2 \sin^2\theta}. \tag{7}$$

The trajectory of a freely moving particle in this space-time may be found in general form from the standard set of equations of geodesic line. For simplicity and due to spherical symmetry we consider the trajectory of the particle in the plane $\theta = \frac{\pi}{2}$, assuming that at time $t = 0$ the particle is in the closest distance ρ from monopole's core, which is by definition, the impact parameter. The trajectory has the following form

$$r = \sqrt{\rho^2 + v^2t^2},\ \ \varphi = \frac{1}{\alpha}\arctan\frac{vt}{\rho},\ \ \theta = \frac{\pi}{2},\ \ u^0 \equiv \gamma = \frac{1}{\sqrt{1 - v^2}}, \tag{8}$$

where v is the constant velocity of the particle and u^0 is the zero component of the four velocity.

To find the total energy radiated by the particle during all its history we adopt an approach used in Ref. [11]. Let us summarize here its main aspects. The total energy radiated by a particle is expressed in terms of the covariant divergence of the energy-momentum tensor as follows

$$\mathcal{E} = \int T^{\nu}_{\mu;\nu}\xi^{\mu}\sqrt{-g(x)}d^4x, \tag{9}$$

where ξ^{μ} is the time-like Killing vector. Taking into account the explicit form of energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{4\pi}\left(\Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\Phi_{,\alpha}\Phi^{,\alpha}\right), \tag{10}$$

the equation of motion for a minimally coupled scalar field (6) and the explicit expression for the Killing vector $\xi^{\mu} = (1, 0, 0, 0)$, one has the following expression for the total energy radiated by the particle during all time

$$\mathcal{E} = 4\pi\int \frac{\partial}{\partial t}D^{rad}(x; x')j(x)j(x')\sqrt{-g(x)}\sqrt{-g(x')}d^4xd^4x', \tag{11}$$

where

$$D^{rad}(x; x') = \frac{1}{2}\left[D^{ret}(x; x') - D^{adv}(x; x')\right] \tag{12}$$

is the radiative Green function. Here the retarded and advanced Green's functions $D^{ret}_{adv}(x; x')$ obey the equation

$$\Box D^{ret}_{adv}(x; x') = -\delta^4(x, x'). \tag{13}$$

In order to find the Green's functions, let us first of all obtain the complete set of eigenfunctions of the Klein-Gordon equation (6)

$$\Box\Phi = \left\{\partial_t^2 - \frac{1}{r^2}\partial_r(r^2\partial_r) + \frac{1}{\alpha^2r^2}\hat{L}^2\right\}\Phi = \lambda^2\Phi, \tag{14}$$

with eigenvalues λ^2. Here $\hat{L}^2$ is the square of the angular momentum operator. The complete set of solution of the equation (14) was considered in the context of quantum fields in Ref. [15], and it has the following form

$$\Phi_{l,m,\omega,p}(t, r, \theta, \varphi) = e^{-i\omega t}\sqrt{\frac{p}{2\pi\alpha^2 r}}J_{\nu_l}(pr)Y_l^m(\theta, \varphi), \tag{15}$$

where $J_{\nu}(x)$ is the Bessel function of first kind; $Y_l^m(\theta, \varphi)$ is the spherical function ($l = 0, 1, 2, \cdots,\ |m| \le l$)); $p = \sqrt{\lambda^2 + \omega^2}$ and

$$\nu_l = \sqrt{\frac{l(l+1)}{\alpha^2} - \frac{1}{4}}. \tag{16}$$

This set of solutions obeys the following relations of orthogonality and completeness

$$\int_0^\infty r^2 dr \int_{-\infty}^{+\infty} dt \int \alpha^2 d\Omega \Phi_{l,m,\omega,p}(x)\Phi^*_{l',m',\omega',p'}(x) = \delta_{l,l'}\delta_{m,m'}\delta(\omega-\omega')\delta(p-p'), \tag{17}$$
$$\sum_{l=0}^\infty \sum_{m=-l}^{+l} \int_{-\infty}^{+\infty} d\omega \int_0^\infty dp \Phi_{l,m,\omega,p}(x)\Phi^*_{l,m,\omega,p}(x') = \delta(t-t')\frac{\delta(r-r')}{\alpha^2 r^2}\frac{\delta(\theta-\theta')\delta(\varphi-\varphi')}{\sin^2\theta}.$$

Using this set of solutions we may represent the retarded and advanced solutions in the following form

$$D^{ret}_{adv}(x;x') = \sum_{l=0}^\infty \sum_{m=-l}^{+l} \int_{-\infty}^{+\infty} d\omega \int_0^\infty dp \frac{\Phi_{l,m,\omega,p}(x)\Phi^*_{l,m,\omega,p}(x')}{p^2-\omega^2 \mp i0} \tag{18}$$

and therefore, the radiative Green function reads

$$D^{rad}(x;x') = \frac{i}{2\alpha^2}\frac{1}{\sqrt{rr'}} \sum_{l=0}^\infty \sum_{m=-l}^{+l} Y_l^m(\theta,\varphi)Y_l^{m*}(\theta',\varphi') \int_{-\infty}^{+\infty} d\omega \ \mathrm{sgn}(\omega) e^{-i\omega(t-t')} \tag{19}$$
$$\times \int_0^\infty dp p J_{\nu_l}(pr) J_{\nu_l}(pr')\delta(p^2-\omega^2).$$

Taking into account this formula into Eq. (11), we obtain the following expression for the total energy

$$\mathcal{E} = \frac{2\pi q^2}{\gamma^2\alpha^2} \sum_{l=0}^\infty \sum_{m=-l}^{+l} \left|Y_l^m(\frac{\pi}{2},0)\right|^2 \int_{-\infty}^{+\infty} d\omega \ |\omega| \int_0^\infty dp p \delta(p^2-\omega^2) \left|S_l^m(\omega,p,v,\rho)\right|^2, \tag{20}$$

where we have introduced the function S_l^m by the relation

$$S_l^m(\omega,p,v,\rho) = \int_{-\infty}^{+\infty} dt e^{i\omega t - i\frac{m}{\alpha}\arctan\frac{vt}{\rho}} \frac{J_{\nu_l}(p\sqrt{\rho^2+v^2t^2})}{(\rho^2+v^2t^2)^{1/4}}. \tag{21}$$

This function obeys the following symmetry relation

$$S_l^m(-\omega,p,v,\rho) = S_l^{-m}(\omega,p,v,\rho). \tag{22}$$

Using this we may represent the total energy as an integral

$$\mathcal{E} = \int_0^\infty d\omega \frac{d\mathcal{E}}{d\omega}, \tag{23}$$

where the spectral density is

$$\frac{d\mathcal{E}}{d\omega} = \omega \frac{2\pi q^2}{\gamma^2\alpha^2} \sum_{l=0}^\infty \sum_{m=-l}^{+l} \left|Y_l^m(\frac{\pi}{2},0)\right|^2 \left|S_l^m(\omega,\omega,v,\rho)\right|^2. \tag{24}$$

The function $S_l^m(\omega,\omega,v,\rho)$ given by Eq. (21) may be represented in a slightly different form, more suitable for analysis (here we assume $\omega > 0$) as

$$S_l^m(\omega,\omega,v,\rho) = -2\frac{\sqrt{\rho}}{v}\sin\frac{\pi}{2}\left[\nu_l - \frac{m}{\alpha} - \frac{1}{2}\right]\tilde{S}_l^m(\omega,v,\rho), \tag{25}$$

where

$$\tilde{S}_l^m(\omega,v,\rho) = \int_1^\infty dy e^{-\frac{\omega\rho}{v}y}\left(\frac{y-1}{y+1}\right)^{-\frac{m}{2\alpha}} \frac{I_{\nu_l}(\omega\rho\sqrt{y^2-1})}{(y^2-1)^{1/4}}. \tag{26}$$

Therefore we can express the spectral density of radiation by

$$\frac{d\mathcal{E}}{d\omega} = \omega\rho\frac{8\pi q^2}{v^2\gamma^2\alpha^2}\sum_{l=0}^{\infty}\sum_{m=-l}^{+l}\left|Y_l^m(\frac{\pi}{2},0)\right|^2\left|\tilde{S}_l^m(\omega,v,\rho)\right|^2\sin^2\frac{\pi}{2}\left[\nu_l-\frac{m}{\alpha}-\frac{1}{2}\right]. \tag{27}$$

Integrating over the frequency ω, using formula 6.612(3) from Ref. [16], we find that the total energy is

$$\mathcal{E} = -\frac{8q^2}{v^3\gamma^2\alpha^2\rho}\sum_{l=0}^{\infty}\sum_{m=-l}^{+l}\left|Y_l^m(\frac{\pi}{2},0)\right|^2\sin^2\frac{\pi}{2}\left[\nu_l-\frac{m}{\alpha}-\frac{1}{2}\right] \tag{28}$$
$$\times\int_1^\infty\frac{dy}{y^2-1}\left(\frac{y-1}{y+1}\right)^{-\frac{m}{2\alpha}}\int_1^\infty\frac{dy'}{y'^2-1}\left(\frac{y'-1}{y'+1}\right)^{-\frac{m}{2\alpha}}(y+y')Q'_{\nu_l-\frac{1}{2}}[\cosh\sigma].$$

Here $Q_\nu[x]$ is the Legendre function of second kind; the prime means the derivative with respect to its argument, and

$$\cosh\sigma = \frac{(y+y')^2v^{-2}-y^2-y'^2+2}{2\sqrt{y^2-1}\sqrt{y'^2-1}}.$$

Now, let us analyze the above expressions in the Minkowski limit. In this case we have to put $\alpha = 1$. Due to this, the argument of sine is $\frac{\pi}{2}(l-m)$. Next we have to take into account that $Y_l^m(\frac{\pi}{2},0) = 0$, if $(l+m)$ is odd. For this reason the argument of sine is $\frac{\pi}{2}\times$ (even number) which implies that the sine of this quantity is zero and as a consequence the total energy is zero, too, as it must be in Minkowski space-time. Differently from the cosmic string space-time there is no specific values of α for which total energy is identically zero.

Let us simplify our formulas for the global monopole space-time assuming that solid angle deficit is small. In this case we can expand sine in the previous formulas in terms of α as

$$\sin^2\frac{\pi}{2}\left[\nu_l-\frac{m}{\alpha}-\frac{1}{2}\right]\approx(1-\alpha)^2\frac{\pi^2}{4}\left[\frac{l(l+1)}{l+\frac{1}{2}}-m\right]^2. \tag{29}$$

Therefore up to $(1-\alpha)^2$ we may set $\alpha = 1$ in the rest part. Firstly, let us analyze the total energy given by Eq. (28). The sum over m can be made using the addition theorem for Legendre function of the first kind from Ref. [16] and results in

$$\mathcal{E} = -\frac{\pi q^2(1-\alpha)^2}{v^3\gamma^2\rho}\int_1^\infty\frac{dy}{y^2-1}\int_1^\infty\frac{dy'}{y'^2-1}(y+y')\sum_{l=0}^{\infty}\left[\left(l+\frac{1}{2}\right)^3-\frac{1}{2}\left(l+\frac{1}{2}\right)+\frac{1}{16}\frac{1}{l+\frac{1}{2}}\right. \tag{30}$$
$$\left.+\ 2\left(\left(l+\frac{1}{2}\right)^2-\frac{1}{4}\right)\partial_\beta+\left(l+\frac{1}{2}\right)\partial_\beta^2\right]P_l[\cosh\beta]Q'_l[\cosh\sigma],$$

where

$$\cosh\beta = \frac{yy'+1}{\sqrt{y^2-1}\sqrt{y'^2-1}}. \tag{31}$$

Using now an integral representation for the Legendre function of the second kind as below

$$Q_l[\cosh\sigma] = \frac{1}{\sqrt{2}}\int_\sigma^\infty\frac{e^{-(l+1/2)t}dt}{\sqrt{\cosh t-\cosh\sigma}}, \tag{32}$$

and the relation

$$\sum_{l=0}^{\infty}e^{-(l+1/2)t}P_l[\cosh\beta] = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{\cosh t-\cosh\beta}}, \tag{33}$$

we get, finally, the following expression for the total energy

$$\mathcal{E} = -(1-\alpha)^2\frac{\pi q^2v\gamma^2}{2\rho}\int_1^\infty\int_1^\infty\frac{dydy'}{(y+y')^3}E(s,y,y'), \tag{34}$$

where

$$E = 1 - 48s^2 z_1 - 192 s^4 z_2^2 - 16 s^2 z_2 \int_0^\infty \frac{dx}{\sqrt{x}} \frac{\partial}{\partial x} \left[\frac{1}{\sqrt{R_x}} \left\{ \frac{1}{(x+1)^{3/2}} + \frac{6s^2 z_1}{(x+1)^{5/2}} + \frac{15}{2} \frac{s^4 z_2^2}{(x+1)^{7/2}} \right\} \right]$$
$$+ \frac{1}{4} \int_0^\infty \frac{dx}{\sqrt{x}} \frac{\partial}{\partial x} \left[\frac{1}{\sqrt{R_x}} \int_x^\infty \frac{dx'}{\sqrt{R_{x'}}} \frac{1}{\sqrt{x'+1}} \right], \tag{35}$$

and we have introduced the following definitions

$$z_1 = \frac{yy'+1}{(y+y')^2}, \; z_2 = \frac{1}{y+y'},$$
$$R_x = (x+1)^2 + 4s^2 z_1 (x+1) + 4 s^4 z_2^2,$$

with the parameter s given by $s = v\gamma$.

The formula obtained looks very awesome but it may be analyzed without great problem for non-relativistic and ultra-relativistic particles. The function E depends only on the combination $s = v\gamma = v/\sqrt{1-v^2}$. In the non-relativistic case the parameter $s \to 0$ and in the ultra-relativistic case $s \to \infty$, and therefore we have to analyze the function E in these two limits.

In the non-relativistic case we may expand all integrands in Eq. (35) over small values of the parameter s and calculate the integrals. The main contribution, in this case, is proportional to the cube of the velocity

$$\mathcal{E} = (1-\alpha)^2 \frac{\pi q^2}{2\rho} v^3. \tag{36}$$

The ultra-relativistic case is more complicate due to the last term in Eq. (35). There is no need, in fact, to calculate the contribution from it. It is enough to find an upper bound for it. Let us analyze the contribution from this term which is given by

$$W = -(1-\alpha)^2 \frac{\pi q^2 v \gamma^2}{8\rho} \int_1^\infty \int_1^\infty \frac{dy dy'}{(y+y')^3} \int_0^\infty \frac{dx}{\sqrt{x}} \frac{\partial}{\partial x} \left[\frac{1}{\sqrt{R_x}} \int_x^\infty \frac{dx'}{\sqrt{R_{x'}}} \frac{1}{\sqrt{x'+1}} \right]. \tag{37}$$

First of all one represents the polynomial R in the following form

$$R_x = (x + 1 + s^2 \delta_+^2)(x + 1 + s^2 \delta_-^2), \tag{38}$$

where $\delta_\pm^2 = 2(z_1 \pm \sqrt{z_1^2 - z_2^2})$. Because $\delta_\pm^2$ are positive we can write out the following inequalities

$$\int_x^\infty \frac{dx'}{\sqrt{R_{x'}}} \frac{1}{\sqrt{x'+1}} \leq \int_0^\infty \frac{dx'}{\sqrt{R_{x'}}} \frac{1}{\sqrt{x'+1}} \leq \int_0^\infty \frac{dx'}{(x+1)^{3/2}} \leq 2. \tag{39}$$

Using this upper bound we have

$$|W| \leq (1-\alpha)^2 \frac{\pi q^2 v \gamma^2}{4\rho} \int_1^\infty \int_1^\infty \frac{dy dy'}{(y+y')^3} \frac{\mathbf{E}(\sqrt{1-\frac{b^2}{a^2}})}{ab^2} \leq (1-\alpha)^2 \frac{\pi^2 q^2 v \gamma^2}{8\rho} \int_1^\infty \int_1^\infty \frac{dy dy'}{(y+y')^3} \frac{1}{ab^2}, \tag{40}$$

where $a = \sqrt{1 + s^2 \delta_+^2}$, $b = \sqrt{1 + s^2 \delta_-^2}$ and we used the fact that the upper bound for elliptic integral of second kind $\mathbf{E}$ is $\pi/2$. Now we change the variables $y \to sy$, $y' \to sy'$ and take the ultra-relativistic limit $s \to \infty$. In the end we have the following estimation

$$|W| \leq (1-\alpha)^2 \frac{\pi^3 q^2}{32\rho}. \tag{41}$$

The contribution of others terms in Eq. (35) is of order larger than γ^3. Calculating the other integrals in Eq. (35) one has that the total energy radiated in ultra-relativistic case is given by

$$\mathcal{E} = (1-\alpha)^2 \frac{3\pi^3 q^2}{32\rho} \gamma^3. \tag{42}$$

Let us consider the spectral density of energy given by Eq. (27) in the case of small solid angle deficit where $|1-\alpha| \ll 1$. First of all we change the variable of integration in Eq. (23) according to $\omega \to \Omega : \ \Omega = \omega\rho/v$ and use the small solid angle deficit approximation for sine given by Eq. (29). Thus, we obtain

$$\frac{d\mathcal{E}}{d\Omega} = \Omega(1-\alpha)^2 \frac{2\pi^3 q^2}{\rho\gamma^2} \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} \left| Y_l^m(\frac{\pi}{2}, 0) \right|^2 \left[\frac{l(l+1)}{l+1/2} - m \right]^2 \left| \tilde{S}_l^m \right|^2 . \tag{43}$$

Now, let us analyze this spectral density in the non-relativistic limit, that is, $v \ll 1$. To do this, we shift the variable of integration in Eq. (26): $x \to x+1$ and use the series expansion for Bessel function in integrand. Thus, we obtain a power series over the velocity. Indeed,

$$\tilde{S}_m^l = e^{-\Omega} \sum_{k=0}^{\infty} \frac{(\frac{\Omega v}{2})^{l+k+1/2}}{k!\Gamma(l+k+3/2)} \int_0^{\infty} dx e^{-\Omega x} x^{\frac{l-m}{2}+\frac{k}{2}} (x+2)^{\frac{l+m}{2}+\frac{k}{2}} . \tag{44}$$

It is easy to see that the lowest orbital momentum, will give the main contribution to the energy of radiation. The zero orbital momentum, $l=0$, does not give contribution to the spectral density (43) because in this case $m=0$. The main contribution comes from the term $l=1$, $(m=\pm 1)$. Taking into account only this orbital momentum we arrive at the formula

$$\frac{d\mathcal{E}}{d\omega} = (1-\alpha)^2 \frac{\pi q^2 v^2}{54} e^{-2\rho\omega/v} \left(49 + (1 + 2\frac{\rho\omega}{v})^2 \right) . \tag{45}$$

It is worth to note that the main contribution to energy comes from the following interval of frequency

$$0 \ \leq \omega \leq \frac{v}{2\rho}, \tag{46}$$

which is due to the exponent in Eq. (44). The maximum of energy is at zero frequency. Integrating Eq. (45) we recover the formula for total energy in the non-relativistic case (36).

For the ultra-relativistic case, the expression for the spectral density is more complicate. We may estimate the interval of frequency from the analysis of the integrand for $\tilde{S}_m^l$:

$$\tilde{S}_m^l = \int_1^{\infty} dy e^{-\Omega y} \left(\frac{y+1}{y-1} \right)^{-\frac{m}{2}} \frac{I_{l+1/2}(\Omega v \sqrt{y^2-1})}{(y^2-1)^{1/4}} . \tag{47}$$

In this case $v \to 1$ and as a consequence all interval of integration in Eq. (47) is important. Because the Bessel function tends exponentially to infinity for large argument, one has the following exponent in the integrand

$$e^{-\Omega(y - v\sqrt{y^2-1})},$$

whose maximum contribution comes from the minimum of the function $y - v\sqrt{y^2-1}$. The minimum of this function is at the point $y_* = \gamma$ and the main contribution from the exponent due to this point is

$$e^{-\Omega/\gamma}.$$

Taking into account this estimation we observe that in the ultra-relativistic case the domain of frequencies is

$$0 \leq \omega \leq \frac{\gamma}{2\rho}. \tag{48}$$

It is worthy noticing that the intervals of frequencies in both cases, non-relativistic and ultra-relativist, are exactly the same that corresponding to the cosmic string space-time.

III. CONCLUSION

We investigated the radiation emitted by a scalar particle moving along a geodesic line in the point-like global monopole space-time. This emission of radiation arises due to the geometric and to the topological features of this space-time. Considering the case of a scalar field minimally coupled with gravity and a specific situation in which the solid angle deficit is small we find that the total energy radiated by a particle along its trajectory is proportional to

the cube of the velocity and to the cube of the Lorenz parameter in the non-relativistic and ultra- relativistic cases, respectively

The spectral density of radiation in the non-relativistic case has a maximum at zero frequency with upper bound $v/2\rho$ and for the ultra-relativistic case, the upper bound is $\gamma/2\rho$.

Let us now compare the energy of the radiation emitted by a uniformly moving charged scalar particle with the one due to the self-force. The self-energy, U_s, of a charged scalar particle at the rest in a global point-like monopole space-time is given by

$$U_s = \frac{q^2}{4r} S(\alpha),$$

which was obtained dividing by two the corresponding result in [10], due to the fact that in the present case we are considering a charged scalar particle.

For small solid angle deficit, the function $S(\alpha) \approx \frac{\pi}{8}(1-\alpha)$ and therefore

$$U_s \approx \frac{\pi q^2}{32r}(1-\alpha).$$

This potential has the same form as the Coulomb interaction (repulsion) between two particles with charges $e_1 = q$ and $e_2 = \pi q(1-\alpha)/32$. Because the potential is proportional to the small parameter $(1-\alpha)$ we can consider, approximately, the particle as moving in the flat Minkowsky space-time and we may use standard results for energy radiation of the Coulomb scattering problem.

For an ultra-relativistic particle (see §73 in Ref. [17]) we have the following expression for the total emitted energy

$$\mathcal{E}_s^{ultra} = \frac{\pi^3}{2048} \frac{q^6(1-\alpha)^2\gamma^2}{m^2\rho^3}. \tag{49}$$

We would like to call attention to the fact that this energy depends on the mass of the particle and this is connected with the fact that the trajectory of particle is not a geodesic.

Let us now compare results in (49) with (42), as follows

$$\frac{\mathcal{E}_s^{ultra}}{\mathcal{E}^{ultra}} = \frac{1}{192}\frac{1}{\rho_c^2\gamma},$$

where $\rho_c = \rho/\lambda_c$ is the impact parameter ρ measured in Compton wavelengths of the scalar particle $\lambda_c = q^2/m$. The classical electrodynamics which we are using is valid for distances much greater than the Compton wavelength, that is, for $\rho_c \gg 1$. Therefore, due to this fact and because $\gamma \gg 1$, we have that

$$\mathcal{E}_s^{ultra} \ll \mathcal{E}^{ultra}.$$

which means that the bremstrahlung of an ultra-relativistic particle prevails over the radiation due to the self-interaction.

In the non-relativistic case (see §70 in Ref. [17]) we have the following ratio

$$\frac{\mathcal{E}_s^{non}}{\mathcal{E}^{non}} = \frac{f(p)}{3\pi(1-\alpha)^2}, \tag{50}$$

where

$$\begin{aligned} f(p) &= p^2\left[(\pi - 2\arctan p)(1+3p^2) - 6p\right], \\ p &= \frac{1-\alpha}{v^2\rho_c}. \end{aligned}$$

The function $f(p)$ tends to zero as πp^2 and to infinity as $8/15p$. It has maximum at $p \approx 1$ with value $f(1) \approx 0.3$.

The ratio (50) may be both greater and smaller then unit. Formal limits of small velocities $v \to 0$ $(p \to \infty)$ or great value of impact parameters $\rho_c \to \infty$ $(p \to 0)$ show that the ratio (50) tends to zero and in these regions the bremsstrahlung prevails over the radiation due to self-interaction. Nevertheless there are some ranges of the velocities and the impact parameter in which the radiation due to the self-interaction prevails over that coming from the bremsstrahlung. For $(1-\alpha) \ll v^2 \ll 1$, we have $p \ll 1$ for arbitrary $\rho_c > 1$ and thus

$$\frac{\mathcal{E}_s^{non}}{\mathcal{E}^{non}} \approx \frac{1}{3v^4\rho_c^2}.$$

Therefore, this ratio is greater than unit if the impact parameter assumes values in the interval

$$1 \ll \rho_c \ll \frac{1}{\sqrt{3}v^2}.$$

As a conclusion we can say that particles moving along geodesic lines in the space-time of a point-like global monopole will emit radiation in the same way as in case of an infinitely thin cosmic string space-time [11]. As in the case of an infinitely thin cosmic string space-time, the energy emitted depends on the angle deficit and vanishes when this angle deficit vanishes, but in the present case, this radiation arises associated with the curvature and non-trivial topology of the space-time of the global monopole, differently from the cosmic string case in which the effect comes exclusively from the non-trivial topology of the space-time.

We have focused our attention not on astrophysical implications of the obtained results but on the features of the global monopole on the radiation phenomena. Nevertheless, the application of these results may have significance from the astrophysical point of view because this radiation mechanism could give rise large energy releases under some conditions relevant in an astrophysical scenario.

Finally, it is interesting to call attention to the fact that based on our results we can obtain the ones corresponding to an electrically charged particle if we just multiply these by two in order to take into account the different polarizations of photons.

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[1] T.W.B. Kibble, J. Phys. A **9**, 1387 (1976).
[2] A. Vilenkin and E.P.S. Shelard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 1994).
[3] W.A. Hiscock, Phys. Rev. Lett. **64**, 344 (1990).
[4] D.P. Bennet and S.H. Rhie, Phys. Rev. Lett. **65**, 1709 (1990).
[5] M. Yamaguchi, Phys. Rev. D **64**, 081301 (2001).
[6] A.E. Lange *et. al.,* Phys. Rev. D **63**, 042001 (2001).
[7] S. Hanany *et al.,* Astrophys. J. Lett. **545**, L5 (2000).
[8] F.R. Bouchet, P. Peter, A. Riazuelo and M. Sakellariadou, Phys. Rev. D **65**, 021301 (2001).
[9] B. Linet, Phys. Rev. D **33**, 1833 (1986); A.G. Smith, in *Formation and Evolution of Cosmic Strings*, edited by G.W. Gibbons, S.W. Hawking, and T. Vachaspati (Cambridge University Press, Cambridge, England, 1990).
[10] E.R. Bezerra de Mello and C. Furtado, Phys. Rev. D **56**, 1345 (1997).
[11] A.N. Aliev and D.V. Gal'tsov, Ann. Phys. **193**, 142 (1989).
[12] E.M. Serebryany, V.D. Skarzhinsky and V.P. Frolov, *Trudy FIAN,* **197**, 166 (1989).
[13] J. Audretsch, U. Jasper and V.D. Skarzhinsky, Phys. Rev. D **49**, 6576 (1994).
[14] M. Barriola and A. Vilenkin, Phys. Rev. Lett., **63**, 341 (1989).
[15] F.D. Mazzitelli and C.O. Lousto, Phys. Rev. D **43**, 468 (1991).
[16] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York, 1980).
[17] L.D. Landau and E.M. Lifshits, *Classical Theory of Fields* (Pergamon Press, Oxford, 1979).